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Improved version of Bohr's inequality

Version améliorée de l'inégalité de Bohr

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ABSTRACT

In this article, we prove several different improved versions of the classical Bohr's inequality. All the results are proved to be sharp.

RÉSUMÉ

Nous montrons ici plusieurs améliorations de l'inégalité de Bohr classique. Nous montrons également que les constantes numériques dans nos résultats sont optimales.

1. Introduction and main results

The classical theorem of Bohr [3] (after subsequent improvements due to M. Riesz, I. Schur and F. Wiener) states that if f is a bounded analytic function on the unit disk $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$, with the Taylor expansion $\sum_{k=0}^{\infty} a_k z^k$, and $\|f\|_\infty := \sup_{z\in\mathbb{D}} |f(z)| < \infty$, then

$$M_f(r) := \sum_{n=0}^{\infty} |a_n| r^n \le \|f\|_\infty \text{ for } 0 \le r \le 1/3 \tag{1}$$

and the constant $1/3$ is sharp. There are a number of articles that deal with Bohr's phenomenon. See, for example, [2,10], the recent survey on this topic by Abu-Muhanna et al. [1] and the references therein. Bombieri [4] considered the function $m(r)$ defined by $m(r) = \sup\left\{M_f(r)/\|f\|_\infty\right\}$, where the supremum is taken over all nonzero bounded analytic functions, and proved that

$$m(r) = \frac{3 - \sqrt{8(1-r^2)}}{r} \text{ for } 1/3 \le r \le 1/\sqrt{2}.$$

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